

## NON-PERTURBATIVE CHIRAL CORRECTIONS FOR LATTICE QCD

A. W. THOMAS, D. B. LEINWEBER

*Department of Physics and Mathematical Physics and  
 Special Research Centre for the Subatomic Structure of Matter  
 University of Adelaide  
 Adelaide SA 5005, AUSTRALIA  
 E-mail: athomas,dleinweb@physics.adelaide.edu.au*

D. H. LU

*Department of Physics, National Taiwan University,  
 Taipei, TAIWAN  
 E-mail: dhlu@phys5.phys.ntu.edu.tw*

We explore the chiral aspects of extrapolation of observables calculated within lattice QCD, using the nucleon magnetic moments as an example. Our analysis shows that the biggest effects of chiral dynamics occur for quark masses corresponding to a pion mass below 600 MeV. In this limited range chiral perturbation theory is not rapidly convergent, but we can develop some understanding of the behaviour through chiral quark models. This model dependent analysis leads us to a simple Padé approximant which builds in both the limits  $m_\pi \rightarrow 0$  and  $m_\pi \rightarrow \infty$  correctly and permits a consistent, model independent extrapolation to the physical pion mass which should be extremely reliable.

### 1 Introduction

At present the only known way to calculate the properties of QCD directly is through the formulation of lattice gauge theory on a discrete space-time lattice. While the lattice formulation of QCD is well established<sup>1</sup>, there have recently been a number of exciting advances in lattice action improvement which are greatly facilitating the reduction of systematic uncertainties associated with the finite lattice volume and the finite lattice spacing. However, direct simulation of QCD for light current quark masses, near the chiral limit, remains computationally intensive. In particular, present lattice calculations of masses in full QCD are limited to equivalent pion masses of order 500 MeV or more – although there have been recent preliminary results from CP-PACS using improved actions at a mass as low as 300 MeV<sup>2</sup>. For nucleon magnetic moments the situation is much worse, with the best calculations still being made within quenched QCD for pion masses greater than 600 MeV. This is clearly beyond the range of validity of chiral perturbation theory ( $\chi$ PT).

In view of this situation, the present approach of calculating the properties of QCD using quark masses away from the chiral regime and extrapolating to the physical world is likely to persist for the foreseeable future. It is therefore vital to understand the quark mass dependence of hadronic observables calculated on the lattice and how to connect these calculations with the physical world. A major difficulty in this endeavour is the rapid rise of the pseudoscalar mass for small increases in the quark mass away from the chiral limit – with  $m_\pi^2 \propto \bar{m}$ , where  $\bar{m}$  is the light quark mass.

Historically, lattice results were often linearly extrapolated with respect to  $m_\pi^2$ , particularly in exploratory calculations. More recently the focus has turned to chiral perturbation theory ( $\chi$ PT), which provides predictions for the leading nonanalytic quark-mass dependence of observables in terms of phenomenological parameters<sup>4,5</sup>. Indeed, it is now relatively standard to use  $\chi$ PT in the extrapolation of lattice simulation data for hadron masses and decay constants<sup>1</sup>. On the other hand, earlier attempts to apply  $\chi$ PT predictions for the quark-mass dependence of baryon magnetic moments failed, as the higher order terms of the chiral expansion quickly dominate the truncated expansion as one moves away from the chiral limit. To one meson loop,  $\chi$ PT expresses the nucleon magnetic moments as<sup>6</sup>

$$\mu_N = \mu_0 + c_1 m_\pi + c_2 m_\pi^2 \log m_\pi + c_3 m_\pi^2 + \cdots, \quad (1)$$

where  $\mu_0$  and  $c_3$  are fitted phenomenologically and  $c_1$  and  $c_2$  are predicted by  $\chi$ PT. The  $m_\pi^2 \log m_\pi$  term quickly dominates as  $m_\pi$  moves away from the chiral limit, making contact with the lattice results impossible.

As a result of these early difficulties, lattice QCD results for baryon magnetic moments<sup>7,8,9</sup> remain predominantly based on linear quark mass (or  $m_\pi^2$ ) extrapolations of the moments expressed in natural magnetons. This approach systematically underestimates the measured moments by 10 to 20%. Finite lattice volume and spacing errors are expected to be some source of systematic error. However,  $\chi$ PT clearly indicates the linear extrapolation of the simulation results is also suspect. It is therefore imperative to find a method which can bridge the void between the realm of  $\chi$ PT and lattice simulations.

We report such a method, which provides predictions for the quark mass (or  $m_\pi^2$ ) dependence of nucleon magnetic moments well beyond the chiral limit – for more details we refer to Ref.<sup>10</sup>. The method is motivated by studies based on the cloudy bag model (CBM), a chiral quark model which preserves the correct leading non-analytic behaviour of chiral perturbation theory while providing what should be a fairly reliable transition to the regime of large pion mass. These studies suggest a Padé approximant which incorporates both the leading nonanalytic structure of  $\chi$ PT and Dirac-moment mass dependence in

the heavy quark-mass regime. We apply this method to the existing lattice data as an illustration of how important such an approach will be to the analysis of future lattice QCD calculations.

## 2 The Cloudy Bag Model

The linearized CBM Lagrangian, with pseudoscalar pion-quark coupling (to order  $1/f_\pi$ ), is given by<sup>11,12</sup>

$$\mathcal{L} = [\bar{q}(i\gamma^\mu\partial_\mu - m_q)q - B]\theta_V - \frac{1}{2}\bar{q}q\delta_S + \frac{1}{2}(\partial_\mu\pi)^2 - \frac{1}{2}m_\pi^2\pi^2 - \frac{i}{2f_\pi}\bar{q}\gamma_5\tau\cdot\pi q\delta_S, \quad (2)$$

where  $B$  is the bag constant,  $f_\pi$  is the  $\pi$  decay constant,  $\theta_V$  is a step function (unity inside the bag volume and vanishing outside) and  $\delta_S$  is a surface delta function. In a lowest order perturbative treatment of the pion field, the quark wave function is not affected by the pion field and is simply given by the MIT bag model. Our calculation is carried out in the Breit frame with the center-of-mass correction for the bag performed via Peierls-Thouless projection. The detailed formulas for calculating nucleon electromagnetic form factors in the CBM are given in Ref.<sup>13</sup>.

In the CBM, a baryon is viewed as a superposition of a bare quark core and bag plus meson states. Both the quark core and the meson cloud contribute to the baryon magnetic moments. These two sources are balanced around a bag radius,  $R = 0.7 - 1.1$  fm<sup>14</sup>. A large bag radius suppresses the contributions from the pion cloud, and enhances the contribution from the quark core. The CBM reproduces the leading non-analytic behavior of  $\chi$ PT, which has as its origin the process shown in Fig. 1(c), along with the other contributions in the model.

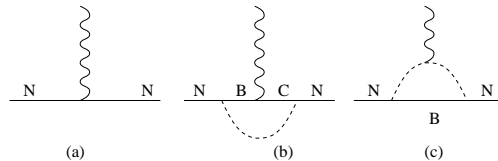


Figure 1: Schematic illustration of the processes included in the CBM calculation.  $B$  and  $C$  denote intermediate state baryons and include  $N$  and  $\Delta$ .

For the  $\pi NN$  vertex, we take a phenomenological, monopole form,  $u(k) = (\Lambda^2 - \mu^2)/(\Lambda^2 + k^2)$ , where  $k$  is the loop momentum and  $\Lambda$  is a cut-off parameter.

As current lattice simulations indicate that  $m_\pi^2$  is approximately proportional to  $m_q$  over a wide range of quark masses<sup>15</sup>, we scale the mass of the quark confined in the bag as  $m_q = (m_\pi/\mu)^2 m_q^{(0)}$ , with  $m_q^{(0)}$  being the current quark mass corresponding to the physical pion mass ( $\mu$ ). In order to obtain a first idea of the behaviour within the model between the lowest mass lattice point and the experimental data, the parameters  $R_0$ ,  $\Lambda$  and  $m_q^{(0)}$  were tuned to reproduce the experimental moment while accommodating the lattice data. It turns out that the best fits are obtained with  $m_q^{(0)}$  in the range 6 to 7 MeV for a bag radius of 0.8fm and  $\Lambda$  of order 600-700 MeV, which are all quite satisfactory.

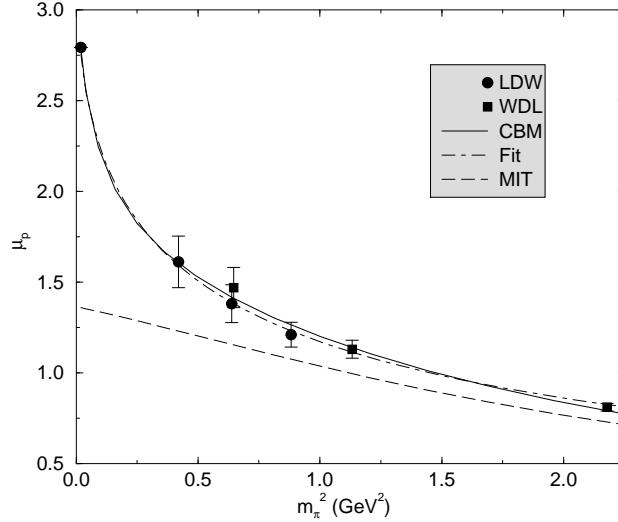


Figure 2: The proton magnetic moment as calculated in lattice QCD (● LDW Ref.<sup>7</sup>, ■ WDL Ref.<sup>8</sup>), the cloudy bag model (CBM) and the MIT bag model (MIT). Also illustrated is a fit of the simple analytic form given in Eq. (3) to the CBM results. The point at the physical value of  $m_\pi^2$  is the experimental measurement and is used to constrain the parameters of the CBM.

Inspection of the results of this calculation, in Fig. 2, show clearly that the pion cloud contribution to the nucleon magnetic moments decreases very quickly, becoming quite small for large quark masses – especially in the range corresponding to the current lattice calculations. In particular, the total pionic correction at the first lattice data point (for  $m_\pi$  around 600 MeV) is of the order of only 10-15% of the total. Since the present lattice data is based on a quenched calculation, which gives incorrect chiral contributions, this is quite

good news. It suggests that in using this data as the basis for a correct chiral extrapolation to the physical mass quenching should not induce a major error.

It is also clear from Fig. 2 that below  $m_\pi = 600$  MeV the behaviour of the magnetic moment is highly non-linear. One clearly needs to account for such behaviour in a reliable manner if we are to make believable extrapolations of the lattice data. Rather than rely on a model dependent extrapolation method, we chose to investigate whether it was possible to find a simple phenomenological form, *with a sound physical basis*, which could do the job. The successful conclusion to that search is described next.

### 3 Encapsulating Formula

After considerable effort we found that the following simple Padé approximant, which builds in both the linear behaviour in  $m_\pi$  as  $m_\pi \rightarrow 0$  (i.e., a square root branch point in  $\bar{m}$ ) and the Dirac moment for large  $m_\pi^2$  (i.e.,  $\mu_N \sim 1/\bar{m}$ ), was able to reproduce the behaviour found in the CBM calculations:

$$\mu_N(m_\pi) = \frac{\mu_N^{(0)}}{1 + \frac{\alpha}{\mu_N^{(0)}} m_\pi + \beta m_\pi^2}. \quad (3)$$

The fit for the proton case is also shown in Fig. 2. Even more remarkable is that the fit parameter  $\alpha$  turned out to be quite close to the value required by  $\chi$ PT (e.g.,  $\alpha = 4.54$  for the proton, compared with  $\alpha = 4.41$  in  $\chi$ PT).

This result encourages us to propose that Eq.(3), with  $\alpha$  taken from  $\chi$ PT, should be used as the method of extrapolation in future analyses of lattice data for nucleon magnetic moments – and suitably generalized, for all members of the baryon octet. As an example, we show in Fig.3 the result of two parameter fits to the proton and neutron data. The nucleon magnetic moments at the physical pion mass, obtained from this extrapolation, are

$$\mu_p = 2.85(22) \mu_N \quad \text{and} \quad \mu_n = -1.90(15) \mu_N, \quad (4)$$

which agree surprisingly well with the experimental measurements, 2.793 and  $-1.913 \mu_N$  respectively. We note that the data required to do a fit of the lattice results in which covariances are taken into account is no longer available. As such, the uncertainties quoted here should be regarded as indicative only.

### 4 Conclusion

We have explored the quark mass dependence of nucleon magnetic moments. Quark masses beyond the region appropriate to chiral perturbation theory

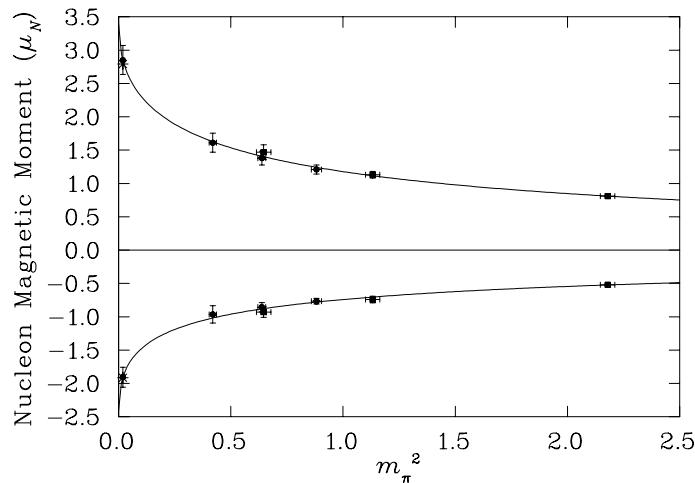


Figure 3: Extrapolation of lattice QCD magnetic moments for the proton (upper) and neutron (lower) to the chiral limit. The curves illustrate a two parameter fit of Eq. (3) to the simulation data in which the one-loop corrected chiral coefficient of  $m_\pi$  is taken from  $\chi$ PT. The experimentally measured moments are indicated by asterisks.

have been explored using the cloudy bag model (CBM) which reproduces the leading nonanalytic behaviour of  $\chi$ PT while modeling the internal structure of the hadron under investigation. We find that the predictions of the CBM are succinctly described by a simple formula which reproduces the leading nonanalytic behavior of  $\chi$ PT in the limit  $m_\pi \rightarrow 0$  and provides the anticipated Dirac moment behavior in the limit  $m_\pi \rightarrow \infty$ . As an example we applied this encapsulating formula to the existing lattice data for nucleon magnetic moments, leading to surprisingly accurate predictions in comparison with the observed values. It will be interesting to see how the fit parameters change as finite volume and lattice spacing artifacts are eliminated in future simulations and whether the level of agreement seen in this investigation is maintained or improved. We strongly advocate the use of the Padé approximant given in Eq. (3) in future lattice QCD investigations of octet baryon magnetic moments.

### Acknowledgements

This work was supported by the Australian Research Council.

### References

1. *Lattice '98*, Proc. of the International Symposium, edited by T. DeGrand, C. DeTar, R. Sugar, and D. Toussaint, Boulder, CO, 1998, Nucl. Phys. B (Proc. Suppl. ), (1998).
2. S. Aoki et al. (CP-PACS), hep-lat/9902018.
3. D. B. Leinweber and T. D. Cohen, Phys. Rev. D **47**, 2147 (1993), hep-lat/9211058; A. W. Thomas, Australian J. Phys. **44**, 173 (1991).
4. M. F. L. Golterman, Acta Phys. Polon. B **25**, 1731 (1994).
5. J. N. Labrenz and S. R. Sharpe, Nucl. Phys. B (Proc. Suppl. ) **34**, 335 (1994).
6. E. Jenkins, M. Luke, A. V. Manohar, and M. J. Savage, Phys. Lett. B **302**, 482 (1993), *ibid.* B **388**, E866 (1996).
7. D. B. Leinweber, R. M. Woloshyn, and T. Draper, Phys. Rev. D **43**, 1659 (1991).
8. W. Wilcox, T. Draper, and K. F. Liu, Phys. Rev. D **46**, 1109 (1992).
9. S. J. Dong, K. F. Liu, and A. G. Williams, Phys. Rev. D **58** 074504 (1998).
10. D. B. Leinweber et al., hep-lat/9810005.
11. S. Théberge, A. W. Thomas and G. A. Miller, *Phys. Rev. D* **22** (1980) 2838; *D* **23** (1981) 2106(E).
12. A. W. Thomas, in *Advances in Nuclear Physics, Vol. 13 (1984) 1*, edited by J. Negele and E. Vogt, New York, 1984, Plenum Press; G. A. Miller, in *Int. Rev. Nucl. Phys.* **2** (1984) 90.
13. D. H. Lu, A. W. Thomas, and A. G. Williams, Phys. Rev. C **57**, 2628 (1998).
14. S. Théberge and A. W. Thomas, Nucl. Phys. A **393** (1983) 252.
15. S. Aoki et al., CP-PACS results for quenched QCD spectrum with the Wilson action, in *Lattice '97*, Proc. of XVth International Symposium on Lattice Field Theory, edited by C. T. H. Davies et al., Edinburgh, UK 1997, Nucl. Phys. B (Proc. Suppl. ), (1998), hep-lat/9710056.